

SOLUTIONS TO THE CHIMP AND BANANAS PROBLEM

A chimp sitting in a space ship moving at $.8c$ eats one banana every five seconds as measured by a Rolex on the chimp's wrist (it is a very rich chimp).

An observer in a stationary space ship uses his set of synchronized clocks and a "chimp banana-eating counter" to measure the chimp's banana consumption as the chimp's ship passes by.

If the observer watches for 15 seconds as measured by his clocks, how many bananas will the chimp eat in that period?

The time relationship between the two frames looks like:

$$\begin{aligned} t_{\text{outsidedlock}} &= \gamma t_{\text{insidedlock}} \\ &= \frac{1}{\left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2}} t_{\text{insidedlock}} \end{aligned}$$

Point #1: I can never remember which subscript is associated with the “fixed” frame outside the ship and which is associated with the “moving” frame inside the ship (I’ve put quotes around the words because someone inside the ship will think the ship is stationary and everything outside is moving, which is to say that everything is relative--for the sake of this problem, though, I’ll stay with “fixed” and “moving” to mean outside and inside).

This memory lapse doesn't matter, though, because I know that time in my frame will always move more quickly than will be the case for time in the other frame. As the relativistic factor is

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

is always bigger than one and as a number larger than one must always multiply the the smaller time. I know the smaller of the two times has to be $t_{\text{insidedlock}}$. This means the relationship must be:

$$\begin{aligned} t_{\text{outsidedlock}} &= \frac{1}{\left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2}} (t_{\text{insidedlock}}) \\ &= \gamma (t_{\text{insidedlock}}) \end{aligned}$$

Point #2: The key to this problem is to determine how many chimp seconds pass while 15 of your seconds pass. To determine that, we can use the time dilation equation thusly:

$$t_{\text{outsidedlock}} = \frac{t_{\text{insidedlock}}}{\left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2}}$$
$$\Rightarrow (15 \text{ seconds}) = \frac{t_{\text{insidedlock}}}{\left(1 - \left(\frac{.8c}{c}\right)^2\right)^{1/2}}$$
$$\Rightarrow (15 \text{ seconds}) = \frac{t_{\text{insidedlock}}}{\left(1 - (.8)^2\right)^{1/2}}$$
$$\Rightarrow (15 \text{ seconds}) = \frac{t_{\text{insidedlock}}}{(1 - .64)^{1/2}}$$
$$\Rightarrow (15 \text{ seconds}) = \frac{t_{\text{insidedlock}}}{(.36)^{1/2}}$$
$$\Rightarrow (15 \text{ seconds}) = \frac{t_{\text{insidedlock}}}{(.6)}$$
$$\Rightarrow (9 \text{ seconds}) = t_{\text{insidedlock}}$$

With 9 seconds of chimp time passing during 15 seconds of your time, it means the chimp will eat

$$(9 \text{ seconds}) \left(\frac{1 \text{ banana}}{5 \text{ seconds}} \right) = 1.8 \text{ bananas}$$